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The Stokes resistance of a spherical cap to translational and rotational motions in a linear shear flow

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Brenner's general results for the force and torque experienced by an arbitrary particle simultaneously translating and rotating in a linear shear flow are applied to the spherical cap. The cap's five fundamental resistance tensors are determined and the corresponding tensors for the sphere and circular disk are recovered as special cases. The problem of a freely moving cap in a linear shear is considered and the resulting translational and rotational motions are analysed. The cap's centre of free rotation is found and trajectories of this point are plotted in a few instances. The cap is also found to possess a 'point of planar motion' which always moves in a plane perpendicular to the vorticity vector of the undisturbed shear regardless of the initial orientation of the cap. It is shown that the motion of the cap actually serves as a model for the motions of all 'oblate' asymmetric bodies of revolution which are moving freely in a linear shear.

1. Introduction

The resistance experienced by an arbitrary particle undergoing translational and rotational motions in a Stokesian flow field is a problem considered by Brenner (1963, 1964*a, b, c*) in a series of four papers. In the first two a scheme is developed for calculating the hydrodynamic force and torque acting on the particle in a fluid at rest at infinity. Parts III and IV extend the analysis to more general flow conditions at infinity.

The formulae derived by Brenner for the force and torque involve tensors which are intrinsic to the particle in question. The components of these tensors depend on the geometry of the particle as well as on the point about which the torque is calculated. The laws by which the tensors transform from point to point are provided by Brenner.

In this paper the general analysis is applied to a particular shape: the spherical cap. More specifically, the problem of a translating rotating spherical cap in a linear shear flow is considered and the five fundamental tensors required to describe the Stokes resistance completely are found. Two parameters determine the cap's geometry: the radius a of the sphere on which the cap lies and the semi-angle α which the rim of the cap subtends at the origin (figure 1). Thus the tensors at any point are functions of the cap radius a and cap angle α ($0 < \alpha \leqslant \pi$). The corresponding tensors for the sphere and circular disk are recovered when the appropriate limits are taken: $\alpha \to \pi$ (sphere); $\alpha \to 0$, $a \to \infty$ with $\alpha a = c$ (circular disk of radius c).

The only bodies for which all five tensors are explicitly known are the ellipsoid and its various special cases (Brenner 1964*b*). Brenner (1964*b, d*) has also examined the slightly deformed sphere and found its corresponding resistance tensors to the first order of deformation. The spherical cap, however, has a geometry quite different from the ellipsoid in that it lacks fore–aft symmetry. This fact makes the determination of the cap's five resistance tensors interesting in its own right; but in addition such asymmetry has been shown by Brenner (1972) to have a major effect on the translational motion of a body which is moving freely in a linear shear flow. Most neutrally buoyant bodies of revolution undergo periodic rotational motion in a simple shear regardless of their other symmetry properties. For those possessing fore–aft symmetry this rotation occurs about the geometrical centre, which simultaneously translates unidirectionally along a streamline. For bodies with no such symmetry however, the rotational motion occurs about the centre of free rotation, which Nir & Acrivos (1973) have found in the case of the spherical dumbbell (two unequal spheres fused at their point of tangency). At the same time this point undergoes a sinuous motion in which it translates periodically back and forth across a streamline. Such bodies suffer no net displacement across the streamlines but their motion is nevertheless in marked contrast to that of axisymmetric bodies possessing fore–aft symmetry.

Nir & Acrivos (1973) seem to be the only authors who have tackled the problem of explicitly solving for the motion of an asymmetric body of revolution in a linear shear flow. They did so without the benefit of Brenner's tensors but their results are consistent with his theory. In this paper Brenner's results will be relied upon heavily in order to determine explicitly the resistance tensors for the spherical cap and then this work will be used to analyse the motion of a neutrally buoyant cap in a linear shear. The results obtained will be consistent with the work of Nir & Acrivos and the cap's centre of free rotation will be found. Another fixed point on the cap's axis whose trajectory is particularly simple to calculate is the point of planar motion, which always moves in a plane perpendicular to the vorticity vector of the undisturbed shear regardless of the cap's orientation. Surprisingly, this point is also independent of the shear causing the motion. The paper concludes with arguments showing that all bodies of revolution have a centre of free rotation and a point of planar motion and that for symmetric bodies these both coincide with the geometrical centre. A method for calculating these points using the components of the resistance tensors is provided.

2. A translating spherical cap in a quiescent fluid

Consider a spherical cap which translates in a viscous fluid at rest at infinity. Let $OXYZ$ be a co-ordinate system attached to the cap such that the origin O is the centre of the sphere $r = a$ on which the cap lies and the z axis is the axis of symmetry (figure 1). Let $\mathbf{U}$ be the velocity of the cap with respect to some fixed co-ordinate system $O'X'Y'Z'$ having the same orientation as $OXYZ$. The moving cap induces a fluid motion which is described by the fluid velocity vector $\mathbf{v}(x',y',z',t)$ referred to the space-fixed system. The incompressible fluid has density ρ and dynamic viscosity μ and the Reynolds number of the motion is assumed to be zero. The cap velocity $\mathbf{U}$ may depend on time but the time scale is assumed to be large enough that the unsteady term $\rho\, d\mathbf{v}/dt$ in the time-dependent Stokes equation is negligible when compared with the viscous term. The governing equations are therefore

$$\nabla p = \mu\nabla^2\mathbf{v}, \quad \nabla.\mathbf{v} = 0, \tag{2.1}$$

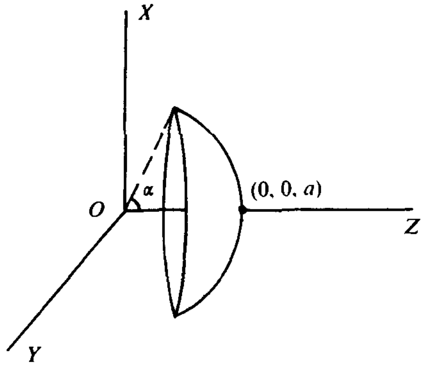

FIGURE 1. The spherical cap.

where p is the fluid pressure and the differential operators are taken with respect to the space-fixed system.

From Brenner (1964*a*) the drag $\mathbf{F}$ on the cap is

$$\mathbf{F} = -\mu \mathbf{K}.\mathbf{U}, \tag{2.2}$$

where $\mathbf{K}$ is a dyadic (second-rank tensor) called the translation tensor. For all particles $\mathbf{K}$ is symmetric and depends solely on geometry. Because the cap is a body of revolution symmetric about the z axis, $\mathbf{K}$ has the form

$$\mathbf{K} = K(\hat{\mathbf{i}}\hat{\mathbf{i}}+\hat{\mathbf{j}}\hat{\mathbf{j}})+K_s\hat{\mathbf{k}}\hat{\mathbf{k}}, \tag{2.3}$$

or equivalently, in terms of a 3×3 matrix

$$\mathbf{K} = \begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & K_s \end{bmatrix}. \tag{2.4}$$

The dot product of the tensor $\mathbf{K}$ and vector $\mathbf{U}$ can be regarded as multiplication of a matrix on the right by a column vector, or in dyadic notation

$$\mathbf{K}.\mathbf{U} = \hat{\mathbf{i}}(\hat{\mathbf{i}}.\mathbf{U})K+\hat{\mathbf{j}}(\hat{\mathbf{j}}.\mathbf{U})K+\hat{\mathbf{k}}(\hat{\mathbf{k}}.\mathbf{U})K_s. \tag{2.5}$$

Thus if the cap moves along its axis, $\mathbf{U} = |\mathbf{U}|\hat{\mathbf{k}}' = |\mathbf{U}|\hat{\mathbf{k}}$ (because the two systems have the same orientation) and $\mathbf{F} = -\mu K_s|\mathbf{U}|\hat{\mathbf{k}}$. The component K_s is the drag coefficient for motion parallel to the axis of symmetry and Collins (1963) has shown this to be

$$K_s = a(6\alpha+8\sin\alpha+\sin 2\alpha). \tag{2.6}$$

Similarly the component K is the drag coefficient for motion perpendicular to the axis of the cap. Dorrepaal (1976) has shown that

$$K = a\left\{6(\alpha+\sin\alpha)-\frac{8}{3}\frac{\sin^2\alpha\cos^4\frac{1}{2}\alpha}{\alpha+\sin\alpha}\right\}. \tag{2.7}$$

If the cap moves parallel to its axis it experiences no torque. Motion in any other direction, however, tends to rotate the cap and from Brenner (1964*a*) the torque $\mathbf{T}_O$ about the origin O of the cap-fixed system is

$$\mathbf{T}_O = -\mu \mathbf{C}_O.\mathbf{U}. \tag{2.8}$$

The dyadic $\mathbf{C}_O$ is a function of geometry and position and is called the coupling tensor at the point O. Since the cap is a body of revolution with O a point on the axis of symmetry, $\mathbf{C}_O$ has the form

$$\mathbf{C}_O = C(\hat{\mathbf{j}}\hat{\mathbf{i}} - \hat{\mathbf{i}}\hat{\mathbf{j}}). \tag{2.9}$$

The component C can be calculated by considering a cap translating with velocity $\mathbf{U} = \hat{\mathbf{i}}' = \hat{\mathbf{i}}$ in a quiescent fluid for then the torque experienced by the cap is

$$\mathbf{T}_O = -\mu C\hat{\mathbf{j}}. \tag{2.10}$$

Ranger (1973) and Dorrepaal (1976) have considered the equivalent problem of a stationary cap placed asymmetrically in a uniform stream and found that

$$C = a^2\left\{8\sin\alpha\cos^2\tfrac{1}{2}\alpha + \frac{16}{3}\frac{\sin^2\alpha\cos^4\frac{1}{2}\alpha}{\alpha+\sin\alpha}\right\}. \tag{2.11}$$

Brenner (1964a) has calculated how the coupling tensor transforms with position. If $\mathbf{r}_{OA}$ is the position vector of some point A fixed with respect to the cap then

$$\mathbf{C}_A = \mathbf{C}_O - \mathbf{r}_{OA}\times\mathbf{K}. \tag{2.12}$$

If A is on the axis of symmetry, x units from the origin, then $\mathbf{r}_{OA} = x\hat{\mathbf{k}}$ and

$$\mathbf{C}_A = (C - xK)(\hat{\mathbf{j}}\hat{\mathbf{i}} - \hat{\mathbf{i}}\hat{\mathbf{j}}). \tag{2.13}$$

Thus for all spherical caps there exists a point R on the axis such that $\mathbf{C}_R = 0$: the point with co-ordinates $(0, 0, C/K)$. This point, called the centre of reaction, is common to all bodies of revolution and will be discussed in greater detail in §4.

3. A translating rotating spherical cap in a quiescent fluid

Consider a spherical cap which simultaneously translates with velocity $\mathbf{U}$ and rotates with angular velocity $\boldsymbol{\omega}$ in a viscous fluid at rest at infinity. The vectors $\mathbf{U}$ and $\boldsymbol{\omega}$ are referred to the space-fixed system and are allowed to be time dependent with the restriction that the time scale be large. If P is a point on the axis of rotation then the instantaneous velocity of the origin O is

$$\mathbf{U}_O = \mathbf{U} + \mathbf{r}_{OP}\times\boldsymbol{\omega}. \tag{3.1}$$

Brenner (1964a) has shown that the force $\mathbf{F}$ on the cap and the torque $\mathbf{T}_O$ about the origin are

$$\mathbf{F} = -\mu\mathbf{K}.\mathbf{U}_O - \mu\mathbf{C}_O^*.\boldsymbol{\omega}, \tag{3.2}$$

$$\mathbf{T}_O = -\mu\mathbf{C}_O.\mathbf{U}_O - \mu\boldsymbol{\Omega}_O.\boldsymbol{\omega}, \tag{3.3}$$

where the asterisk indicates the transpose.

The only new tensor in the above two expressions is $\boldsymbol{\Omega}_O$, the rotation tensor at O. The subscript indicates that $\boldsymbol{\Omega}_O$ is a function of position as well as geometry, and because the cap is a body of revolution with O on the axis of symmetry, the rotation tensor is of the form

$$\boldsymbol{\Omega}_O = \Omega(\hat{\mathbf{i}}\hat{\mathbf{i}} + \hat{\mathbf{j}}\hat{\mathbf{j}}) + \Omega_s\hat{\mathbf{k}}\hat{\mathbf{k}}. \tag{3.4}$$

The component Ω_s is the torque coefficient for rotational motion about the axis of symmetry. Collins (1959) has solved the problem of a spherical cap rotating about its axis and from his solution

$$\Omega_s = a^3\{8\alpha - 4\sin 2\alpha + \tfrac{16}{3}\sin^3\alpha\}. \tag{3.5}$$

The calculation of $\mathbf{\Omega}$ is more difficult. Suppose the cap is placed such that OY coincides with $O'Y'$. Assume that the cap rotates about the origin with angular velocity $\boldsymbol{\omega} = \hat{\jmath}' = \hat{\jmath}$, but does not translate. The point P in (3.1) is any point along the axis OY and so $\mathbf{U}_O = 0$. From (3.3) the torque about the origin is

$$\mathbf{T}_O = -\mu\Omega\hat{\jmath}. \tag{3.6}$$

Now even though $\boldsymbol{\omega}$ is constant this is a time-dependent problem because of asymmetry. But with the assumption that the angular velocity is small, which is necessary if the Stokes approximation is to be valid, the time dependence can be ignored and the equivalent problem of a stationary cap in a steadily rotating fluid can be used to calculate the component Ω. This problem has been solved by Dorrepaal (1975) by the integral-transform technique invented by Ranger (1973). The torque which the cap experiences leads to the result

$$\Omega = a^3\left\{8(\alpha+\sin\alpha) - \tfrac{8}{3}\sin^3\alpha - \tfrac{32}{3}\frac{\sin^2\alpha\cos^4\frac{1}{2}\alpha}{\alpha+\sin\alpha}\right\}. \tag{3.7}$$

The transformation law for the rotation tensor has been found by Brenner (1964*a*). It is

$$\mathbf{\Omega}_A = \mathbf{\Omega}_O - \mathbf{r}_{OA}\times\mathbf{K}\times\mathbf{r}_{OA} + \mathbf{C}_O\times\mathbf{r}_{OA} - \mathbf{r}_{OA}\times\mathbf{C}_O^*, \tag{3.8}$$

where A is any point fixed with respect to the cap.

4. The centre of hydrodynamic stress

In § 2 it was shown that all caps possess a centre of reaction R at which $\mathbf{C}_R = 0$. The role played by R is comparable to that played by the centre of mass in rigid-body dynamics in that the translational and rotational motions are uncoupled at R. This is clear when (3.2) and (3.3) are expressed in terms of the point R rather than the origin:

$$\mathbf{F} = -\mu\mathbf{K}.\mathbf{U}_R, \quad \mathbf{T}_R = -\mu\mathbf{\Omega}_R.\boldsymbol{\omega}. \tag{4.1}$$

The hydrodynamic force on the cap depends only on the instantaneous translational velocity of R while the torque about R depends solely on the instantaneous angular velocity of the cap. The rotation tensor $\mathbf{\Omega}_R$ at the point R is easily calculated from (3.8) to be

$$\mathbf{\Omega}_R = \Omega_R(\hat{\imath}\hat{\imath}+\hat{\jmath}\hat{\jmath}) + \Omega_s\hat{\mathbf{k}}\hat{\mathbf{k}}, \tag{4.2}$$

where

$$\Omega_R = \Omega - C^2/K. \tag{4.3}$$

It was mentioned earlier that the resistance tensors of the cap's two limiting cases are recovered when α and a are chosen appropriately. When $\alpha = \pi$ the cap becomes the sphere $r = a$ and the centre of reaction R coincides with the centre of the sphere. From (2.6) and (2.7) the translation tensor becomes

$$\mathbf{K} = 6\pi a\mathbf{I}, \tag{4.4}$$

where $\mathbf{I} = \hat{\imath}\hat{\imath}+\hat{\jmath}\hat{\jmath}+\hat{\mathbf{k}}\hat{\mathbf{k}}$ is the dyadic idemfactor, and it is seen immediately that the highly symmetric form of $\mathbf{K}$ reflects the isotropy of the sphere with respect to translation. Using (3.5) and (3.7) a familiar result for the rotation tensor is obtained, viz.

$$\mathbf{\Omega}_R = 8\pi a^3\mathbf{I}. \tag{4.5}$$

On the other hand if $\alpha \to 0$ and $a \to \infty$ such that $a\alpha = c$, the cap becomes a circular disk of radius c. As with the sphere the centre of reaction R coincides with the centre of the disk and this is in fact true for all bodies of revolution which possess fore–aft symmetry. In the case of the disk, the translation tensor simplifies to

$$\mathbf{K} = \tfrac{32}{3}c(\hat{\imath}\hat{\imath}+\hat{\jmath}\hat{\jmath})+16c\hat{\mathbf{k}}\hat{\mathbf{k}}, \tag{4.6}$$

which agrees with known results (Brenner 1963), and the rotation tensor becomes

$$\mathbf{\Omega}_R = \tfrac{32}{3}c^3\mathbf{I}, \tag{4.7}$$

which verifies the isotropy of the disk with respect to rotation.

5. A translating rotating cap in a linear shear flow

Up to now we have considered a moving cap in a quiescent fluid. There are three tensors which describe the resistance experienced by the cap and the components of these tensors have been found in the literature. The problem of a translating rotating cap in a linear shear flow will now be considered and this is a motion which has not been treated before.

The undisturbed linear shear is represented by the vector $\mathbf{u}$, referred to the space-fixed system. In terms of tensors the shear is described by the velocity-gradient dyadic $\mathbf{G} = \nabla\mathbf{u}$, whose components are $G_{ij} = \partial u_j/\partial x_i'$. According to Brenner (1964*b*) the cap experiences a force $\mathbf{F}$ and a torque $\mathbf{T}_O$ about the origin given by

$$\mathbf{F} = -\mu[\mathbf{K}.(\mathbf{U}_O-\mathbf{u}_O)+\mathbf{C}_O^*.(\boldsymbol{\omega}-\boldsymbol{\omega}_f)+\mathbf{\Phi}_O:\mathbf{S}], \tag{5.1}$$

$$\mathbf{T}_O = -\mu[\mathbf{C}_O.(\mathbf{U}_O-\mathbf{u}_O)+\mathbf{\Omega}_O.(\boldsymbol{\omega}-\boldsymbol{\omega}_f)+\boldsymbol{\tau}_O:\mathbf{S}], \tag{5.2}$$

where $\mathbf{u}_O$ is the undisturbed shear flow vector $\mathbf{u}$ evaluated at the origin O, $\boldsymbol{\omega}_f = \frac{1}{2}\operatorname{curl}\mathbf{u}$ is the fluid spin vector of the shear and $\mathbf{S} = \frac{1}{2}(\mathbf{G}+\mathbf{G}^*)$. The triadics $\mathbf{\Phi}_O$ and $\boldsymbol{\tau}_O$ are tensors of rank 3 known as the shear-force and shear-torque tensors at O. The colon separating $\mathbf{\Phi}_O$ from the dyadic $\mathbf{S}$ in (5.1) represents a double dot product. In terms of components this can be written as

$$\mathbf{\Phi}_O:\mathbf{S} = \Phi_{ijk}S_{kj}, \tag{5.3}$$

where repeated indices are summed over 1 to 3.

The terms involving $\mathbf{\Phi}_O$ and $\boldsymbol{\tau}_O$ in (5.1) and (5.2) give the force and torque on the cap resulting from the linear shear. The triadics are functions of geometry and position and are always post-symmetric, i.e.

$$\Phi_{ijk} = \Phi_{ikj}, \quad \tau_{ijk} = \tau_{ikj}. \tag{5.4}$$

In addition these tensors are unique only to within an additive term of the form $\mathbf{a}\mathbf{I}$, where $\mathbf{a}$ is any vector. This is due to the condition of incompressibility and is easily shown by considering the identity $\mathbf{I}:\mathbf{S} = \operatorname{div}\mathbf{u} = 0$. Therefore

$$(\mathbf{\Phi}_O+\mathbf{a}\mathbf{I}):\mathbf{S} = \mathbf{\Phi}_O:\mathbf{S}+\mathbf{a}(\mathbf{I}:\mathbf{S}) = \mathbf{\Phi}_O:\mathbf{S}. \tag{5.5}$$

Because the cap is a body of revolution with O a point on the axis of symmetry, $\mathbf{\Phi}_O$ and $\boldsymbol{\tau}_O$ have the following forms (Brenner 1964*b*):

$$\mathbf{\Phi}_O = \Phi(\hat{\imath}\hat{\imath}\hat{\mathbf{k}}+\hat{\imath}\hat{\mathbf{k}}\hat{\imath}+\hat{\jmath}\hat{\jmath}\hat{\mathbf{k}}+\hat{\jmath}\hat{\mathbf{k}}\hat{\jmath})+\Phi_s\hat{\mathbf{k}}\hat{\mathbf{k}}\hat{\mathbf{k}}, \tag{5.6}$$

$$\boldsymbol{\tau}_O = \tau(\hat{\imath}\hat{\jmath}\hat{\mathbf{k}}+\hat{\imath}\hat{\mathbf{k}}\hat{\jmath}-\hat{\jmath}\hat{\imath}\hat{\mathbf{k}}-\hat{\jmath}\hat{\mathbf{k}}\hat{\imath}). \tag{5.7}$$

The shear-force triadic has only two independent non-zero components and the shear-torque triadic has but one.

The calculation of the components Φ and τ can be achieved by considering a spherical cap at rest in a linear shear. In this case the space-fixed and body-fixed systems can be chosen to coincide. We have $\mathbf{U}_O = \boldsymbol{\omega} = 0$ and if the stagnation plane of the shear is chosen to pass through the origin in figure 1 we also have $\mathbf{u}_O = 0$. This still is an involved problem however because after the force on the cap has been computed, the rotational effects of the shear, which contribute the term $\mu\mathbf{C}^*_O\cdot\boldsymbol{\omega}_f$, must be subtracted off to obtain the required drag contribution.

Fortunately the calculation of Φ and τ can be made more direct by considering a different condition at infinity. If we require that the fluid velocity $\mathbf{v}$ satisfies the condition

$$\mathbf{v} \sim \mathbf{u} = \tfrac{1}{2}S(z\hat{\mathbf{\imath}} + x\hat{\mathbf{k}}) \quad \text{as} \quad (x^2+y^2+z^2)^{\frac{1}{2}} \to \infty, \tag{5.8}$$

where S is some typical velocity gradient, then the flow past the stationary cap is a two-dimensional hyperbolic flow with velocities at infinity of the same order of magnitude as in a linear shear. Once again the origin is a stagnation point of the undisturbed flow ($\mathbf{u}_O = 0$) and now the flow far from the cap is irrotational, giving $\boldsymbol{\omega}_f = 0$. In this problem, therefore, the only contributions to the force and torque come from the terms involving $\boldsymbol{\Phi}_O$ and $\boldsymbol{\tau}_O$ and thus the components Φ and τ can be calculated directly.

From (5.8)

$$\mathbf{G} = \nabla\mathbf{u} = \tfrac{1}{2}S(\hat{\mathbf{\imath}}\hat{\mathbf{k}} + \hat{\mathbf{k}}\hat{\mathbf{\imath}}), \tag{5.9}$$

and because $\mathbf{G}$ is symmetric $\mathbf{S} = \mathbf{G}$. The force and torque on the cap are, from (5.1) and (5.2),

$$\mathbf{F} = -\mu S\Phi\hat{\mathbf{\imath}}, \quad \mathbf{T}_O = \mu S\tau\hat{\mathbf{\jmath}}. \tag{5.10}$$

The two-dimensional hyperbolic flow past a stationary spherical cap can be solved using Ranger's (1973) integral-transform technique. A suitable non-dimensional representation for the fluid velocity is

$$\mathbf{v} = \operatorname{curl}^2\left\{\frac{\psi(r,\theta)}{r\sin\theta}\,\mathbf{r}\cos\phi\right\} + \operatorname{curl}\left\{\frac{V(r,\theta)}{r\sin\theta}\,\mathbf{r}\sin\phi\right\}, \tag{5.11}$$

where (r,θ,ϕ) are spherical polar co-ordinates, and the condition (5.8) at infinity becomes

$$\psi \sim \tfrac{1}{6}r^3\sin^2\theta\cos\theta, \quad V = o(r) \quad \text{as} \quad r\to\infty. \tag{5.12}$$

The method of solution is exactly as in Ranger (1973) and the drag and torque on the cap yield the following results:

$$\Phi = -a^2\left\{\tfrac{20}{3}\sin\alpha\cos^2\tfrac{1}{2}\alpha - \frac{8}{3}\frac{\sin^2\alpha\cos^4\frac{1}{2}\alpha(1-\frac{8}{3}\sin^2\frac{1}{2}\alpha)}{\alpha+\sin\alpha}\right\}, \tag{5.13}$$

$$\tau = a^3\left\{\tfrac{8}{3}\sin\alpha\cos^2\tfrac{1}{2}\alpha(1-6\sin^2\tfrac{1}{2}\alpha) + \frac{16}{3}\frac{\sin^2\alpha\cos^4\frac{1}{2}\alpha(1-\frac{8}{3}\sin^2\frac{1}{2}\alpha)}{\alpha+\sin\alpha}\right\}. \tag{5.14}$$

The only remaining component is Φ_s. This component is not required when calculating the drag in a linear shear flow but it will be needed in later analysis. It too can be found by considering an appropriate flow past a stationary cap whose body-fixed co-ordinate system coincides with the space-fixed system.

An examination of the shear-force triadic in (5.6) reveals that Φ_s affects the drag only in flows where the velocity-gradient dyadic has a non-zero component G_{33}. Thus in the undisturbed flow the z component of velocity must be a function of z and because of incompressibility other diagonal elements of the dyadic $\mathbf{G}$ must also be non-zero. The simplest axisymmetric flow with these features is

$$\mathbf{u} = \tfrac{1}{6}S(x\hat{\mathbf{i}}+y\hat{\mathbf{j}}-2z\hat{\mathbf{k}}). \tag{5.15}$$

This flow is most suitable for our purposes because it is irrotational ($\boldsymbol{\omega}_f = 0$) and because the origin is a stagnation point ($\mathbf{u}_O = 0$). The dyadic describing the flow is

$$\mathbf{G} = \mathbf{S} = \tfrac{1}{6}S(\hat{\mathbf{i}}\hat{\mathbf{i}}+\hat{\mathbf{j}}\hat{\mathbf{j}}-2\hat{\mathbf{k}}\hat{\mathbf{k}}) \tag{5.16}$$

and thus the drag experienced by the cap is

$$\mathbf{F} = -\mu\boldsymbol{\Phi}_O : \mathbf{S} = \tfrac{1}{3}\mu S\Phi_s\hat{\mathbf{k}}. \tag{5.17}$$

Fortunately this problem is also soluble by Ranger's integral transform technique. In this instance an axisymmetric version of the method is used and the details parallel exactly the solution of Dorrepaal, O'Neill & Ranger (1976). The fluid velocity is represented in terms of a stream function $\psi(r,\theta)$ as

$$\mathbf{v} = \operatorname{curl}\left\{-\frac{\psi(r,\theta)}{r\sin\theta}\hat{\boldsymbol{\Phi}}\right\} \tag{5.18}$$

and the condition (5.15) at infinity is

$$\psi \sim \tfrac{1}{6}r^3\sin^2\theta\cos\theta \quad \text{as} \quad r\to\infty. \tag{5.19}$$

For the sake of brevity the details of the solution are omitted. The component Φ_s is found to be

$$\Phi_s = -16a^2\sin\alpha\cos^4\tfrac{1}{2}\alpha. \tag{5.20}$$

Having determined the shear-force and shear-torque tensors at the origin, it is necessary to know how they vary with position. These transformation laws are provided by Brenner (1964*b*) and are repeated here:

$$\boldsymbol{\Phi}_A = \boldsymbol{\Phi}_O+\tfrac{1}{2}[\mathbf{K}\mathbf{r}_{OA}+(\mathbf{K}\mathbf{r}_{OA})^*], \tag{5.21}$$

$$\boldsymbol{\tau}_A = \boldsymbol{\tau}_O+\tfrac{1}{2}[(\mathbf{C}_O-\mathbf{r}_{OA}\times\mathbf{K})\,\mathbf{r}_{OA}+\{(\mathbf{C}_O-\mathbf{r}_{OA}\times\mathbf{K})\,\mathbf{r}_{OA}\}^*]-\mathbf{r}_{OA}\times\boldsymbol{\Phi}_O. \tag{5.22}$$

In particular, at the centre of reaction R,

$$\boldsymbol{\Phi}_R = \Phi_R(\hat{\mathbf{i}}\hat{\mathbf{i}}\hat{\mathbf{k}}+\hat{\mathbf{i}}\hat{\mathbf{k}}\hat{\mathbf{i}}+\hat{\mathbf{j}}\hat{\mathbf{j}}\hat{\mathbf{k}}+\hat{\mathbf{j}}\hat{\mathbf{k}}\hat{\mathbf{j}})+(\Phi_R)_s\hat{\mathbf{k}}\hat{\mathbf{k}}\hat{\mathbf{k}}, \tag{5.23}$$

$$\boldsymbol{\tau}_R = \tau_R(\hat{\mathbf{i}}\hat{\mathbf{j}}\hat{\mathbf{k}}+\hat{\mathbf{i}}\hat{\mathbf{k}}\hat{\mathbf{j}}-\hat{\mathbf{j}}\hat{\mathbf{i}}\hat{\mathbf{k}}-\hat{\mathbf{j}}\hat{\mathbf{k}}\hat{\mathbf{i}}), \tag{5.24}$$

where

$$\Phi_R = \Phi+\tfrac{1}{2}C, \quad (\Phi_R)_s = \Phi_s+K_sC/K, \quad \tau_R = \tau+C\Phi/K. \tag{5.25}$$

For a sphere,

$$\boldsymbol{\Phi}_R = 0, \quad \boldsymbol{\tau}_R = 0. \tag{5.26}$$

For a circular disk of radius c,

$$\boldsymbol{\Phi}_R = 0, \quad \boldsymbol{\tau}_R = -\tfrac{16}{3}c^3(\hat{\mathbf{i}}\hat{\mathbf{j}}\hat{\mathbf{k}}+\hat{\mathbf{i}}\hat{\mathbf{k}}\hat{\mathbf{j}}-\hat{\mathbf{j}}\hat{\mathbf{i}}\hat{\mathbf{k}}-\hat{\mathbf{j}}\hat{\mathbf{k}}\hat{\mathbf{i}}). \tag{5.27}$$

6. A freely moving cap in a linear shear flow

Now that the cap's five resistance tensors have been determined we are in a position to discuss the motion of a neutrally buoyant cap in a linear shear. The hydrodynamic force on the cap vanishes as does the torque about all points E and so from (5.1) and (5.2) the resulting translational and angular velocities are

$$\mathbf{U}_E = \mathbf{u}_E - \mathbf{A}_E : \mathbf{S}, \quad \boldsymbol{\omega} = \boldsymbol{\omega}_f - \mathbf{B} : \mathbf{S}, \tag{6.1), (6.2}$$

where $\mathbf{A}_E$ and $\mathbf{B}$ are respectively the translational and rotational slip-velocity triadics. In terms of the resistance tensors,

$$\mathbf{A}_E = (\mathbf{K} - \mathbf{C}_E^* . \boldsymbol{\Omega}_E^{-1} . \mathbf{C}_E)^{-1} . (\boldsymbol{\Phi}_E - \mathbf{C}_E^* . \boldsymbol{\Omega}_E^{-1} . \boldsymbol{\tau}_E), \tag{6.3}$$

$$\mathbf{B} = (\boldsymbol{\Omega}_E - \mathbf{C}_E . \mathbf{K}^{-1} . \mathbf{C}_E^*)^{-1} . (\boldsymbol{\tau}_E - \mathbf{C}_E . \mathbf{K}^{-1} . \boldsymbol{\Phi}_E). \tag{6.4}$$

Because $\boldsymbol{\omega}$, $\boldsymbol{\omega}_f$ and $\mathbf{S}$ in (6.2) are independent of E, the same must be true of $\mathbf{B}$. A much simpler form for $\mathbf{B}$ can be obtained, therefore, by choosing $E = R$, the centre of reaction, in (6.4):

$$\mathbf{B} = \boldsymbol{\Omega}_R^{-1} . \boldsymbol{\tau}_R = \tfrac{1}{2} B(\hat{\mathbf{\imath}}\hat{\mathbf{\jmath}}\hat{\mathbf{k}} + \hat{\mathbf{\imath}}\hat{\mathbf{k}}\hat{\mathbf{\jmath}} - \hat{\mathbf{\jmath}}\hat{\mathbf{\imath}}\hat{\mathbf{k}} - \hat{\mathbf{\jmath}}\hat{\mathbf{k}}\hat{\mathbf{\imath}}), \tag{6.5}$$

where

$$B = 2\tau_R/\Omega_R. \tag{6.6}$$

For all bodies of revolution the tensor $\mathbf{B}$ has the form shown in (6.5). Bretherton (1962) and Brenner (1972) have shown that the magnitude of the coefficient B plays a major role in determining the particle's resultant motion. In particular, if $|B| < 1$ the effect of the shear is to rotate the particle in such a way that the end of its axis traces out an ellipse. There is an infinite family of such periodic orbits and that actually described depends on the initial orientation of the particle. However, if $|B| \geqslant 1$ the motion is not periodic and the particle assumes a preferred orientation.

In the case of the cap it can be shown from (6.6) that $-1 < B \leqslant 0$ for $0 < \alpha \leqslant \pi$. Thus all spherical caps undergo periodic rotation in a linear shear flow. In the case of the circular disk however, $B = -1$ and from Brenner's (1972) analysis the disk translates edge-on to the flow while rotating about its axis with an angular velocity dependent on its initial orientation.

The sign of the coefficient B indicates something about the geometry of the body. In the case of an ellipsoid of revolution with polar radius $a_\parallel$ and equatorial radius $a_\perp$,

$$B = (a_\parallel^2 - a_\perp^2)/(a_\parallel^2 + a_\perp^2). \tag{6.7}$$

Thus prolate ellipsoids ($a_\parallel > a_\perp$) have positive values of B while oblate ellipsoids ($a_\parallel < a_\perp$) have negative values of B. Even though the spherical cap is unlike the ellipsoid of revolution in that it lacks fore–aft symmetry, the fact that B is negative for all caps indicates the 'oblateness' of the cap's geometry. The spherical dumbbell of Nir & Acrivos (1973) is an example of a prolate body of revolution lacking fore–aft symmetry.

Since the rotational motion of all bodies of revolution in a linear shear has been treated in detail by Brenner (1972), the main purpose of this paper is to focus on the translational motion of the cap. Because the cap is rotating as it translates, the body-fixed co-ordinate system is rotating with respect to the space-fixed system. If $\hat{\mathbf{\imath}}$, $\hat{\mathbf{\jmath}}$ and $\hat{\mathbf{k}}$ are the three unit vectors along the axes fixed in the cap ($\hat{\mathbf{k}}$ being along the cap's axis)

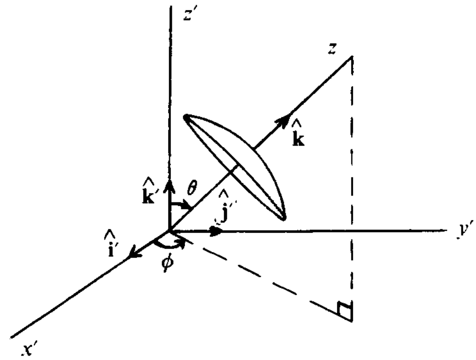

FIGURE 2.

and if $\hat{\imath}'$, $\hat{\jmath}'$ and $\hat{k}'$ are unit vectors along the space-fixed axes, then the following relations exist between these vectors, where θ and ϕ are the angles shown in figure 2:

$$\left.\begin{aligned}\hat{\imath} &= \hat{\imath}'\cos\theta\cos\phi+\hat{\jmath}'\cos\theta\sin\phi-\hat{k}'\sin\theta,\\ \hat{\jmath} &= -\hat{\imath}'\sin\phi+\hat{\jmath}'\cos\phi,\\ \hat{k} &= \hat{\imath}'\sin\theta\cos\phi+\hat{\jmath}'\sin\theta\sin\phi+\hat{k}'\cos\theta.\end{aligned}\right\} \tag{6.8}$$

Now suppose that the cap is suspended in a shear described by

$$\mathbf{u} = Sx'\hat{\jmath}', \tag{6.9}$$

where S is the shear rate. The cap responds by rotating and so θ and ϕ are functions of time. Brenner (1972) has found that

$$\tan\theta = C_1[1+(R_1^2-1)\sin^2\{2\pi(t-t_0)/T\}]^{\frac{1}{2}}, \tag{6.10}$$

$$\tan\phi = R_1\tan\{2\pi(t-t_0)/T\}, \tag{6.11}$$

where

$$R_1 = \left(\frac{1+B}{1-B}\right)^{\frac{1}{2}} \tag{6.12}$$

is the equivalent axis ratio of the cap,

$$T = (2\pi/S)(R_1+R_1^{-1}) \tag{6.13}$$

is the period of the rotation and C_1 and t_0 are constants of integration which can be determined if the initial orientation of the cap is known.

Equation (6.1) determines the translational motion of any point E fixed with respect to the cap. If E lies along the cap's axis then from Brenner (1972) the triadic $\mathbf{A}_E$ has the symmetric form

$$\mathbf{A}_E = A_\parallel\hat{k}\hat{k}\hat{k}+[\mathbf{I}\hat{k}+(\mathbf{I}\hat{k})^*]A_\perp, \tag{6.14}$$

which when expanded becomes

$$\mathbf{A}_E = (A_\parallel+2A_\perp)\hat{k}\hat{k}\hat{k}+A_\perp(\hat{\imath}\hat{\imath}\hat{k}+\hat{\imath}\hat{k}\hat{\imath}+\hat{\jmath}\hat{\jmath}\hat{k}+\hat{\jmath}\hat{k}\hat{\jmath}). \tag{6.15}$$

The components $A_\parallel$ and $A_\perp$ are functions of α, a and E.

Now suppose that the position vector of E with respect to the centre of reaction R is

$$\mathbf{r}_{RE} = \gamma\hat{k}. \tag{6.16}$$

From Brenner (1972) the components $A_{\parallel}$ and $A_{\perp}$ evaluated at the point E are

$$A_{\parallel}(E) = A_{\parallel}(R)+\gamma B, \tag{6.17}$$

$$A_{\perp}(E) = A_{\perp}(R)+\tfrac{1}{2}\gamma(1-B). \tag{6.18}$$

From (6.3),

$$\mathbf{A}_R = \mathbf{K}^{-1}.\mathbf{\Phi}_R = \Phi_R/K(\hat{\imath}\hat{\imath}\hat{\mathbf{k}}+\hat{\imath}\hat{\mathbf{k}}\hat{\imath}+\hat{\jmath}\hat{\jmath}\hat{\mathbf{k}}+\hat{\jmath}\hat{\mathbf{k}}\hat{\jmath})+(\Phi_R)_s/K_s\,\hat{\mathbf{k}}\hat{\mathbf{k}}\hat{\mathbf{k}}. \tag{6.19}$$

It follows then that

$$A_{\perp}(R) = \Phi_R/K, \quad A_{\parallel}(R) = (\Phi_R)_s/K_s-2\Phi_R/K. \tag{6.20}$$

Substituting (6.6) and (6.20) into (6.17) and (6.18), we have

$$A_{\parallel}(E) = (\Phi_R)_s/K_s-2\Phi_R/K+2\gamma\tau_R/\Omega_R, \tag{6.21}$$

$$A_{\perp}(E) = \Phi_R/K+\gamma(\tfrac{1}{2}-\tau_R/\Omega_R). \tag{6.22}$$

Turning now to the linear shear flow (6.9) which causes the motion, the dyadic $\mathbf{S}$ describing this shear is

$$\mathbf{S} = \tfrac{1}{2}S(\hat{\imath}'\hat{\jmath}+\hat{\jmath}'\hat{\imath}'). \tag{6.23}$$

To compute the product $\mathbf{A}_E : \mathbf{S}$ in (6.1) it is necessary to express $\mathbf{A}_E$ in (6.15) in terms of space-fixed co-ordinates using (6.8). When this is done and the product taken we obtain

$$\begin{aligned}\mathbf{A}_E : \mathbf{S} = {} & \hat{\imath}'S[A_{\parallel}(E)\sin^3\theta\sin\phi\cos^2\phi+A_{\perp}(E)\sin\theta\sin\phi] \\ & +\hat{\jmath}'S[A_{\parallel}(E)\sin^3\theta\sin^2\phi\cos\phi+A_{\perp}(E)\sin\theta\cos\phi] \\ & +\hat{\mathbf{k}}'S[A_{\parallel}(E)\sin^2\theta\cos\theta\sin\phi\cos\phi].\end{aligned} \tag{6.24}$$

It is now possible using (6.1) and (6.24) to calculate the trajectory

$$\boldsymbol{\rho}_E(t) = x_E'(t)\,\hat{\imath}'+y_E'(t)\,\hat{\jmath}'+z_E'(t)\,\hat{\mathbf{k}}' \tag{6.25}$$

of any point E on the cap's axis of symmetry. The translational velocity of the point E is

$$\mathbf{U}_E = d\boldsymbol{\rho}_E(t)/dt \tag{6.26}$$

and the fluid velocity of the undisturbed shear at E is

$$\mathbf{u}_E = Sx_E'(t)\,\hat{\jmath}'. \tag{6.27}$$

When these two expressions are combined with (6.24), which is time dependent by virtue of (6.10) and (6.11), a first-order differential equation for the trajectory $\boldsymbol{\rho}_E(t)$ is obtained which may be easily solved numerically.

7. The centre of free rotation

It is obvious from (6.22) that a point Q exists on the cap's axis where $A_{\perp}(Q) = 0$. The position vector of Q with respect to the centre of reaction R is $\mathbf{r}_{RQ} = \gamma\hat{\mathbf{k}}$, where

$$\gamma = \Omega_R\,\Phi_R/K(\tau_R-\tfrac{1}{2}\Omega_R). \tag{7.1}$$

The equations of motion for the point Q are

$$dx_Q'(t)/dt = -SA_{\parallel}(Q)\sin^3\theta\sin\phi\cos^2\phi, \tag{7.2a}$$

$$dy_Q'(t)/dt = S[x_Q'(t)-A_{\parallel}(Q)\sin^3\theta\sin^2\phi\cos\phi], \tag{7.2b}$$

$$dz_Q'(t)/dt = -SA_{\parallel}(Q)\sin^2\theta\cos\theta\sin\phi\cos\phi. \tag{7.2c}$$

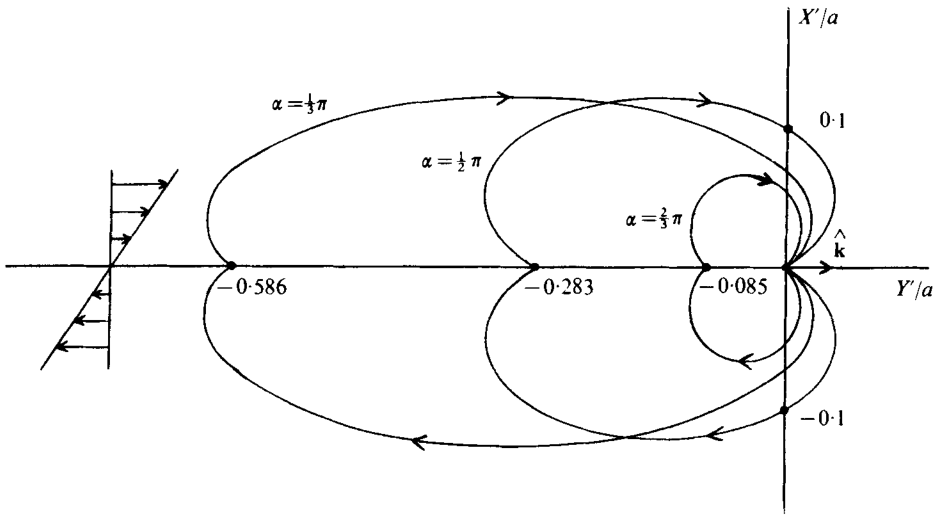

FIGURE 3. The vector $\hat{\mathbf{k}}$ gives the initial orientation of the cap ($\phi_0 = \frac{1}{2}\pi$).

Now since the translational slip-velocity triadic at the point Q has the highly symmetric form

$$\mathbf{A}_Q = A_{\parallel}(Q)\,\hat{\mathbf{k}}\hat{\mathbf{k}}\hat{\mathbf{k}}, \tag{7.3}$$

the product $\mathbf{A}_Q : \mathbf{S}$ is always a vector in the direction of the cap's axis $\hat{\mathbf{k}}$ regardless of the shear. It follows that the velocity $\mathbf{U}_Q$ of the point Q is made up of the velocity $\mathbf{u}_Q$ of the undisturbed flow at Q plus an axial 'drift' velocity $\mathbf{A}_Q : \mathbf{S}$. The latter can in no way be due to the rotational motion of the particle and so Q must be the point on the cap's axis about which the rotational motion takes place. The point Q is consequently called the centre of free rotation. Nir & Acrivos (1973) have derived the equations of motion for the centre of free rotation for the spherical dumbbell and their equations have exactly the same form as (7.2). In fact the only difference between these two systems is that the dumbbell is a prolate body ($R_1 > 1$) while the cap is oblate ($R_1 < 1$). The parameter R_1 enters the equations when θ and ϕ are expressed as functions of time.

Suppose that the cap is initially situated such that $\phi(0) = \phi_0$. Let the constant entering the problem when (7.2a) is integrated be so chosen that $x'_Q(0) = 0$. Nir & Acrivos have shown that when $\phi_0 = \frac{1}{2}\pi$ or $\frac{3}{2}\pi$ the centre of free rotation Q moves in a closed orbit. In other words Q has a closed trajectory if initially the cap's axis lies in the stagnation plane of the shear. For other values of ϕ_0, the point Q experiences a net translation in the direction of flow.

Suppose that the 'orbit constant' C_1 in (6.10) is chosen to be infinite so that $\theta = \frac{1}{2}\pi$ is a constant. This simplifies the trajectory of the centre of free rotation by making it two-dimensional (because (7.2c) reduces to $z'_Q(t) = \text{constant}$). In figure 3 the closed two-dimensional trajectories of Q are plotted for three different caps ($\alpha = \frac{1}{3}\pi, \frac{1}{2}\pi, \frac{2}{3}\pi$). These trajectories are similar to those of the spherical dumbbell's centre of free rotation except that the caps' trajectories are stretched in the direction of flow. When ϕ_0 is given values other than $\frac{1}{2}\pi$ or $\frac{3}{2}\pi$, the trajectories of Q are found to be very similar to the corresponding trajectories of the dumbbell (see Nir & Acrivos 1973) and sketches of these are omitted to avoid duplication.

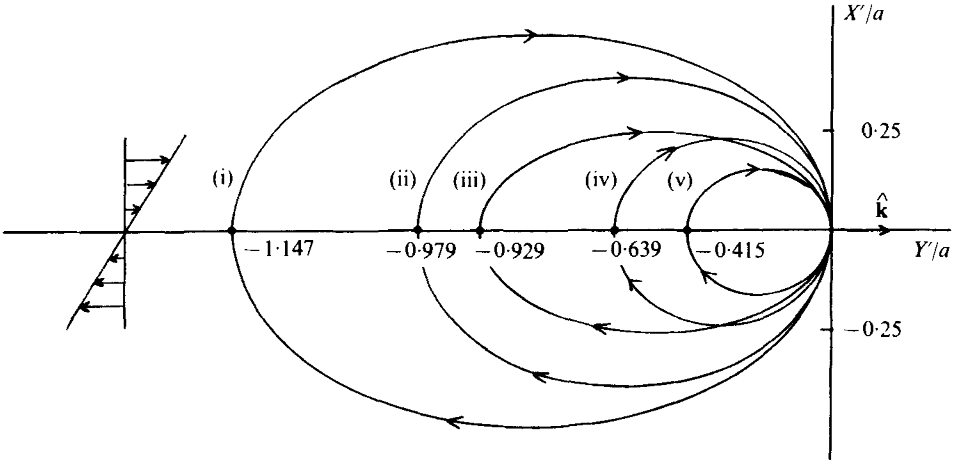

FIGURE 4. In all of these trajectories the cap's initial orientation is $\phi_0 = \frac{1}{2}\pi$. (i) $\alpha = \frac{2}{3}\pi$, $\theta(0) = \frac{1}{2}\pi$. (ii) $\alpha = \frac{1}{2}\pi$, $\theta(0) = \frac{1}{2}\pi$. (iii) $\alpha = \frac{1}{3}\pi$, $\theta(0) = \frac{1}{2}\pi$. (iv) $\alpha = \frac{1}{2}\pi$, $\theta(0) = \frac{1}{4}\pi$. (v) $\alpha = \frac{1}{2}\pi$, $\theta(0) = \frac{1}{6}\pi$.

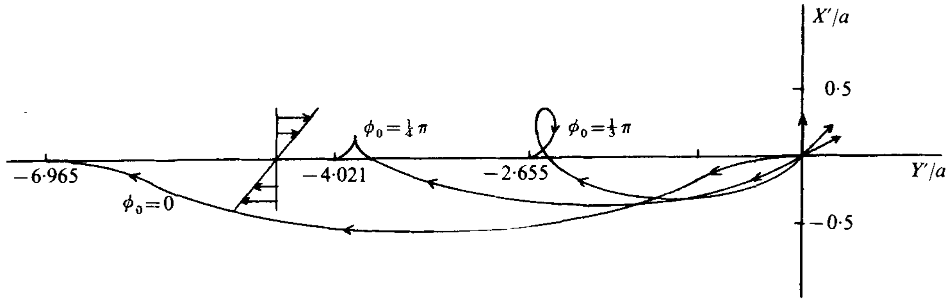

FIGURE 5. The trajectories of the point of planar motion for a hemispherical cap ($\alpha = \frac{1}{2}\pi$) with $\theta(0) = \frac{1}{2}\pi$.

8. The point of planar motion

A point on the cap's axis whose trajectory is particularly simple to calculate is the point G where $A_{\parallel}(G) = 0$. From (6.21) the position vector of G is $\mathbf{r}_{RG} = \gamma\hat{\mathbf{k}}$, where

$$\gamma = \Omega_R[\Phi_R K_s - \tfrac{1}{2}K(\Phi_R)_s]/KK_s\tau_R. \tag{8.1}$$

The equations of motion for the point G are

$$dx'_G(t)/dt = -SA_{\perp}(G)\sin\theta\sin\phi, \tag{8.2a}$$

$$dy'_G(t)/dt = S[x'_G(t) - A_{\perp}(G)\sin\theta\cos\phi], \tag{8.2b}$$

$$dz'_G(t)/dt = 0. \tag{8.2c}$$

It is notable that the trajectory of G is always parallel to the x', y' plane [because of (8.2c)]. Thus for any given cap there exists a point G which always moves in a plane perpendicular to the vorticity vector of the undisturbed shear. This point G is called the 'point of planar motion'.

In figures 4 and 5, trajectories of G are plotted. The cap is situated such that $\phi(0) = \phi_0$ and $x'_G(0) = 0$ and once again the trajectories are closed when $\phi_0 = \frac{1}{2}\pi$ or $\frac{3}{2}\pi$

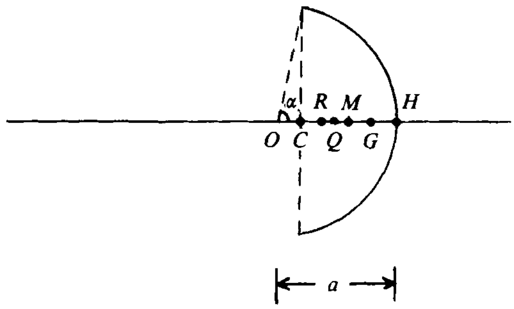

FIGURE 6. Relative positions of R, Q, M and G for a typical spherical cap.

(figure 4). Figure 5 shows the effect on the trajectories of varying the cap's initial orientation ϕ_0.

In figure 6 a typical cap is shown with the relative positions of G, Q, R and M, the cap's centre of mass. The position vector of M is

$$\mathbf{r}_{OM} = \tfrac{1}{2}a(1+\cos\alpha)\,\hat{\mathbf{k}} \tag{8.3}$$

and so M lies midway between C and H. The relative positions of R, Q, M and G are the same for all caps and in the cases of the sphere and circular disk these points all coincide with the geometrical centre.

9. The cap as a model for oblate asymmetric bodies of revolution

Many of the results of the last three sections have much wider application than just to the spherical cap. All bodies of revolution have a centre of reaction R and so the quantities Ω_R, Φ_R, $(\Phi_R)_s$, τ_R as well as K and K_s are meaningful for such bodies. It follows then that all axisymmetric particles possess a centre of free rotation Q defined by (7.1) and a point of planar motion G defined by (8.1), where $\hat{\mathbf{k}}$ is a unit vector in the direction $\mathbf{r}_{RM}$. When the body of revolution has fore–aft symmetry the centres of reaction and mass coincide with the geometrical centre. It is also true that the shear-force tensor of such bodies vanishes at R and so $\Phi_R = (\Phi_R)_s = 0$. From (7.1) and (8.1), therefore, the centre of free rotation Q and the point of planar motion G also coincide with R and M, at the geometrical centre.

The sphere is an example of a symmetric body of revolution whose shear-torque tensor also vanishes at R. Thus $\tau_R = 0$. In such cases $\mathbf{r}_{RG}$, defined in (8.1), does not necessarily tend to zero and in fact in the particular case of the sphere $\mathbf{r}_{RG} = \tfrac{2}{3}a\hat{\mathbf{k}} \neq 0$. But this is immaterial because, from (6.6), $B = 0$ for such a body and therefore it rotates exactly like a particle of fluid. In addition the body has fore–aft symmetry and so its centre simply moves along with the fluid. These two facts taken together mean that all points in the body have planar trajectories and so every point of the particle (in particular its centre) can be called a point of planar motion.

In addition to being an asymmetric body of revolution the spherical cap is oblate because it has a negative value of B ($R_1 < 1$). The equations of motion for the centre of free rotation Q and the point of planar motion G for any geometrically similar body will therefore be qualitatively similar to (7.2) and (8.2) respectively. Thus the trajectories discussed in §§7 and 8 and sketched in figures 3–5 are similar in shape to those traced out by the centre of free rotation and the point of planar motion of any oblate asymmetric body of revolution. In this way the cap serves as a model for the motion of such a body in a linear shear flow. Similarly the spherical dumbbell of Nir & Acrivos serves as a model for the motion of a prolate asymmetric body of revolution in a simple shear.

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